1.5 Long Division & Synthetic Division

Long Division –

<u>Division Algorithm for Polynomials :</u>

Let *f*(*x*) and *d*(*x*) be polynomials with the degree of *f* greater than or equal to the degree of *d*, and *d*(*x*) $\neq 0$. Then there are unique polynomials *q(x)* and *r(x),* called the **quotient** and **remainder**, such that

f(x) = d(x)• q(x) + r(x) (where *d(x)* is the **divisor**)

where either *r(x)* = 0 or the degree of *r* is less than degree of *d*.

If *r(x)*= 0 then *d(x)* divides evenly into *f(x).*

Example:

Use long division to find the quotient and remainder when $2x^4 - x^3 - 2$ is divided by $2x^2 + x + 1$
Write a summary statement in both polynomial and fraction form.

Solution:

$$
\begin{array}{r l}
 & \quad\quad\quad\; x^2 - x \\
2x^2 + x + 1 \big) & 2x^4 - x^3 + 0x^2 + 0x - 2 \\
 & \underline{2x^4 + x^3 + x^2 \quad\quad\quad\quad} \\
 & \quad -2x^3 - x^2 + 0x - 2 \\
 & \quad \underline{-2x^3 - x^2 - x \quad\;\;} \\
 & \quad\quad\quad\quad\quad x - 2 \qquad \leftarrow \text{Remainder}
\end{array}
$$

The division algorithm yields the polynomial form

$$2x^4 - x^3 - 2 = \left(2x^2 + x + 1\right)\left(x^2 - x\right) + \left(x + 2\right).$$

This can also be written as

$$\frac{2x^4 - x^3 - 2}{2x^2 + x + 1} = x^2 - x + \frac{x + 2}{2x^2 + x + 1} \quad \text{(fraction form)}$$

Synthetic Division:

Example:

Divide $2x^3 - 3x^2 - 5x - 12$ by $x - 3$ using synthetic division and write a summary statement in fraction form.

Solution:

The zero of the divisor x – 3 is 3, which we put in the divisor position. Because the dividend is in standard form, we write its coefficients in order in the dividend position, *making sure to use zero as a placeholder for any missing term.* We leave space for the line for products and draw a horizontal line below the space. (See below.)

- Because the leading coefficient of the dividend must be the leading coefficient of the quotient, copy the 2 into the first quotient position.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & & & & \\ \hline & 2 & & & \end{array}$$

- Multiply the zero of the divisor (3) by the most recently determined coefficient of the quotient (2). Write the product above the line and one column to the right.

- Add the next coefficient of the dividend to the product just found and record the sum below the line in the same column.

- Repeat the "multiply" and "add" steps until the last row is completed.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

The last line of numbers are the coefficients of the quotient polynomial and the remainder. The quotient must be a quadratic function. (Why?) So the quotient is $2x^2 + 3x + 4$ and the remainder is 0. So we conclude that $\frac{2x^3 - 3x^2 - 5x - 12}{x - 3} = 2x^2 + 3x + 4,\ x \neq 3.$